Primary submission: 06.06.2017 | Final acceptance: 14.11.2017

How Taxes, Education and Public Capital Influence Economic Growth in Poland

Michał Konopczyński[1]

ABSTRACT

This paper investigates the relationship between economic growth in Poland and selected elements of fiscal policy and private spending on education. We use the Mankiw-Romer-Weil model, augmented with learning-by-doing and spillover-effects and with concepts from the literature on optimal fiscal policy. We demonstrate that, from 2000-2015, economic growth in Poland was primarily driven by rapid improvements in the level of human capital (at 4.4% per annum) coupled with a rapid increase in public capital (6.0%) and secondarily due to the accumulation of private capital (2.1% annually). Simulations of tax cuts suggest that a synchronized reduction of all tax rates by 5 percentage points (pp) in Poland should increase the annual GDP growth rate by approximately 0.32 pp. Increasing (private or public) spending on education by 1 pp of the GDP would increase the growth rate by approximately 0.3 pp. We also analyze the effects of increasing public capital. The stock of public capital in Poland is still below the optimal level, and it may be beneficial to increase investment in public capital at the cost of public consumption (which is intuitively clear) and – to some extent – at the cost of public spending on education.

KEY WORDS: optimal fiscal policy, income taxes, labor taxes, capital taxes, economic growth, human capital, public capital

JEL Classification: E62, H21, H52

[1] Uniwersytet Ekonomiczny w Poznaniu - Katedra Ekonomii Matematycznej, Poland

Introduction

In Konopczyński (2014), we developed a simple exogenous growth model to investigate the long-term effects of modifications in fiscal policy. Our model simulated the long-term effects of changes in tax rates and changes in private and public spending on education. However, the model did not include one important factor of growth that (as we demonstrate in this paper) has been a significant driver of the recent impressive growth of the Polish economy: unprecedented acceleration of public investment, mainly in transportation infrastructure: roads and highways, railways, airports, seaports, etc., financed largely by the EU Cohesion Fund. As Kollias and Paleologou (2013) argue: "a range of other economic activities gain from such public spending. In particular, improved transport infrastructure reduces effective distances between different poles of economic activity, between centres of production and consumption, and reduces road congestion bringing about lower travel times and costs for both enterprises and passengers. Increased trade is a strong stimulus of growth." The benefits in Poland are not limited to internal and international trade. Improving public in-

Correspondence concerning this article should be addressed to: **Michał Konopczyński**, Uniwersytet Ekonomiczny w Poznaniu - Katedra Ekonomii Matematycznej, al. Niepodległości 10, Poznań 61-875, Poland. E-mail: michal.konopczynski@ue.poznan.pl

frastructure attracts foreign direct investment, which is yet another important factor of growth for Poland.

The main purpose of this paper is to augment our previous model by adding public capital. The augmented model allows for simulating the effects of changes in the level of government investment in public capital. We also update the simulations presented in Konopczyński (2014) regarding tax rates and spending on education.

Our description of public capital follows the so-called "stock approach", in which public capital is regarded as an accumulated stock that depreciates over time. This approach was initiated by Arrow & Kurz (1970) and developed by many researchers, with notable contributions by Futagami Morita and Shibata (1993), Easterly & Rebelo (1993), Turnovsky (1997; 2004), Fisher & Turnovsky (1998), Dasgupta (1999) and many others. More recent contributions include Chen (2007), who focuses on the productive role of roads, railways, airports, seaports, and public transportation. In his model, public capital serves as a substitute to private capital. However, Chen does not distinguish human capital from physical (private) capital. In addition, (in Chen's own words) "the government behaves passively (…). It collects both the labor income taxes and capital income taxes in each period, and then spends the total amount of taxation in accumulating public infrastructure stock". He analyses income taxes only – and there is no tax on consumption.

Another interesting contribution is that of Marrero and Novales (2007), who analyze income taxes and expenditures on public (utility-enhancing) consumption and investment in infrastructure. However, they assume that public and private capital "fully depreciates each period", which makes their model difficult to interpret and virtually impossible to verify empirically. Similarly, to Chen (2007), they abstract from human capital and assume a permanently balanced government budget.

Conversely, human capital is explicitly present in the analysis of Dhont and Heylen (2009), who construct a closed economy model with several types of taxes, public consumption and productive government spending, including education spending, active labor market expenditures, R&D expenditures, and public investment. These types of spending are aggregated and accumulated as human capital, which is one factor of production.

Agénor and Yilmaz (2011) examine several alternative fiscal rules in their endogenous growth model. The government spends money on infrastructure (a stock) and healthcare (a flow). They numerically evaluate the performance of fiscal rules. A similar model combining both types of public spending is presented by Bucci and Bo (2011). In their model, public capital is partly used as an input in the production of final output (a flow) and partly accumulated to increase its own supply in the future (a stock). The share of productive government expenditures devoted to production services can be exogenous or endogenous (the government serves as benevolent social planner). They find that, in the second case, the share of public investment in the GDP is an important determinant of the long-term growth rate. However, their model relies on several simplifying assumptions, e.g., there is only one flat tax (implicitly, because they only consider government expenditures as a share of the GDP), there is no depreciation, no technological progress, no human capital.

One of the most recent theoretical contributions that includes human capital and stock and flow approaches is Escobar-Posada and Monteiro (2015). They present a two-sector model of physical and human capital accumulation in which public goods provide both productive capital and utility-enhancing services. They analyze the impacts of the level of government expenditure and its composition on growth and welfare and derive their respective growth and welfare-maximizing levels. The latest paper that combines stock and flow approaches is Zhang Ru and Li (2016). They derive the optimal tax structure and show that it is equivalent to the optimal public spending composition.

Contrary to most of the aforementioned literature, our analysis is deliberately based on a simple exogenous growth theory for several reasons, listed in Konopczyński (2014). In endogenous growth theory, economic agents are constantly optimizing, adjusting savings and consumption in response to policy changes. In our view, it would be overly optimistic to assume that Central and Eastern European Countries (CEEC's) are already in this type of equilibrium. These countries remain in a transition from centrally planned, Eastern-oriented economies to market-based economies integrated with the West (the EU). Moreover, over the last 25 years, the CEEC's economies have undergone deep structural changes and significant (often sudden) modifications in economic systems and policies. External conditions have also rapidly evolved, with the great (revolutionary) change of the expansion of the EU in 2004.

We consider four types of taxes: on capital, labor, human capital and consumption. Tax revenues are expended on public consumption, education, public capital, with the remainder transferred back to households. The government budget is permanently balanced, which is a standard assumption in most research on optimal fiscal policy. This assumption is fully justified for closed economy models by the well-known Ricardian equivalence.

The paper is organized as follows. Section 1 presents the details of our growth model. Section 2 contains a qualitative sensitivity analysis. In section 3, the model is calibrated based on statistical data on the Polish economy from 2000-2015. Section 4 contains the baseline scenario. Sections 5 and 6 present scenarios of tax cuts and increased educational expenditures by the government and private sector. In section 7, we determine the optimal structure of private investment. Section 8 contains simulations of increased spending on public capital. Section 9 completes the investigation, with an analysis of changes in tax rates accommodated by appropriate changes in expenditures on public capital or education. The robustness of the results is briefly discussed in section 10. The summary synthesizes the main results and offers some critical remarks. Mathematical proofs are included in the appendix.

1. The economy with the government investing in public capital

To incorporate public services into the model, the aggregate production function (2) used in Konopczyński (2014) is generalized as follows:

$$Y = aK^{\alpha} H^{1-\alpha-\beta} (EL)^{\beta} P^{\beta}, \quad 0<\alpha,\beta<1 \tag{1}$$

where K denotes physical capital, H represents human capital, L is raw labor, and P represents **public productive services** provided by the government. Equation (1) implies constant returns to scale in the private inputs, K, H and L. Note that the public services P are complimentary with all private inputs, i.e., an increase in P increases their marginal products. Following the standard approach in the literature, we assume that the exponent on P is equal to β (see, e.g., Barro and Martin, 2004, p. 220).

Many public services are subject to congestion, e.g., roads and highways, seaports and airports, communication infrastructure, water provision and other publicly provided utilities, courts, police and fire services. This problem is especially visible in Poland, which is the subject of the empirical analysis in the second part of this paper. Therefore, following the standard approach in the literature initiated by Barro and Martin (1992), we assume that P is a linear function of the stock of public productive capital K_P per unit of the GDP, i.e.,

$$P = b\, K_P / Y, \quad b = const. > 0 \tag{2}$$

Without loss of generality, parameter b can be normalized to 1 because, after substituting equation (2) into (1) and separating b^{β} from $(K_P/Y)^{\beta}$, we can combine a with b^{β} and replace it with a new constant. Thus, we set $b=1$.

Following Romer (1986) and Barro and Martin (2004), we assume positive externalities related to learning-by-doing and spillover-effects, embedded in the labor-augmenting technology index E, which is proportional to the capital per worker ratio, i.e., $E = x K/L$, where $x = const. > 0$. Thus, the production function can be written as

$$Y = AK^{\alpha+\beta} H^{1-\alpha-\beta} P^{\beta}, \tag{3}$$

where $A = ax^{\beta} = const > 0$. Therefore, the aggregate output of the economy is described by a Cobb-Douglas function with constant returns to scale for both types of capital (physical and human). The labor supply in the country is growing exponentially:

$$L = L_0 e^{nt}, \tag{4}$$

where $L_0 > 0$ denotes the initial stock of labor (at $t=0$) and $t \geq 0$ is a continuous time index. Demand for all three factors of production results from the rational decisions of firms maximizing profits in perfectly competitive markets. Let w_K and w_H denote the real rental price of physical capital and human capital, respectively, and let w denote the real wage rate. In the profit maximizing equilibrium, all factors are paid their marginal products, i.e.,

$$MPK = \partial Y/\partial K = \alpha Y/K = w_K = r + \delta_K \, , \tag{5}$$

$$MPH = \partial Y/\partial H = (1-\alpha-\beta)Y/H = w_H \, , \tag{6}$$

$$MPL = \partial Y/\partial L = \beta Y/L = w \, , \tag{7}$$

where δ_K represents the rate of depreciation of capital. Note that the variables w, w_H and $w_K = r + \delta_K$ represent gross rates, i.e., the unit costs of labor, human capital and physical capital from the perspective of the representative firm, respectively.

The public sector (i.e., the government) levies income and consumption taxes. Let τ_L, τ_H, and τ_K denote the average tax rates on labor, human capital and physical capital stock, respectively. Taxes on labor and human capital are levied on gross wage rates, i.e., the government collects $\tau_L w$ and $\tau_H w_H$. The income tax on capital is calculated as follows: $\tau_K(w_K - \delta_K) = \tau_K r$, i.e., the tax is levied on net capital income, defined as gross income minus a depreciation allowance. The total sum of all income taxes is expressed as

$$T_1 = \tau_L wL + \tau_H w_H H + \tau_K rK \, , \tag{8}$$

In addition, the government collects consumption taxes equal to

$$T_2 = \tau_C C \, , \tag{9}$$

where C is the aggregate consumption. The total government revenue is $T = T_1 + T_2$. The government maintains a balanced budget in each period, i.e., $G = T$. This assumption is justified by Ricardian equivalence – see, for example, Elmendorf and Mankiw (1998), and it is commonly applied in the literature; see, for example, Lee and Gordon (2005), Dhont and Heylen (2009), and Turnovsky (2009).

The assumption that Poland maintains a balanced budget may appear unrealistic. Waiving this assumption may significantly change the results and implications of the model presented in this paper. An interested reader may refer to the book by Konopczyński (2015), in which we present a detailed analysis of the generalized version of the model used herein by allowing the government to borrow both internally and from abroad.

Public expenditures include four components:

$$G = G_T + G_E + G_C + G_K \, , \tag{10}$$

where G_T denotes cash transfers to the private sector (social transfers: pensions, various benefits, etc.), G_E represents public spending on education, G_C is public consumption (primarily health care, national defense, and public safety), and G_K denotes public spending on productive capital (e.g., transport and communication infrastructure, public utilities and R&D infrastructure). To assure balanced-growth equilibrium in the model, we must assume that various categories of public spending are proportional to the GDP:

$$G_T = \gamma_T Y \, , \quad \text{where} \quad 0 < \gamma_T < 1 \, . \tag{11}$$

$$G_E = \gamma_E Y \, , \quad \text{where} \quad 0 < \gamma_E < 1 \, . \tag{12}$$

$$G_K = \gamma_P Y \, , \quad \text{where} \quad 0 < \gamma_P < 1 \, . \tag{13}$$

Obviously, $\gamma_T + \gamma_E + \gamma_P < 1$. In a closed economy, the total compensation of all production factors is equal to the output. Therefore, households' disposable income Y_d is equal to the GDP net of taxes, plus transfers. A fraction of that income is saved, and the remainder is consumed; hence, the budget constraint of the private sector is expressed as follows:

$$Y_d = Y - T_1 - T_2 + G_T = C + S \, . \tag{14}$$

We assume a constant, exogenous rate of savings:

$$S = \gamma Y_d = \gamma(Y - T_1 - T_2 + G_T) \, . \tag{15}$$

Savings are invested in physical and human capital, with a fixed share coefficient $0 < \psi < 1$:

$$I_K = (1-\psi)S \, , \tag{16}$$

$$I_H = \psi S \, , \tag{17}$$

From (14), it follows that private consumption is equal to:

$$C = Y_d - S = Y - T_1 - T_2 + G_T - S \, . \tag{18}$$

Notice that equations (15) and (18) are interconnected because of (9). According to (15), savings depend on consumption and, simultaneously, according to (18) consumption depends on savings. For convenience, we solve this system of equations. Simple algebraic manipulation yields:

$$C = A_1(Y - T_1 + G_T), \quad \text{where} \quad A_1 = \frac{1-\gamma}{1+\tau_C(1-\gamma)} \tag{19}$$

$$S = A_2(Y - T_1 + G_T), \quad \text{where} \quad A_2 = \frac{\gamma}{1+\tau_C(1-\gamma)} \tag{20}$$

Henceforth, for simplicity, certain expressions (functions of parameters) are denoted by A_1, A_2, etc. Substituting (8) and (11), and using (5) – (7), equation (20) can be written as:

$$S = A_2\big[(1-\alpha\tau_K - \beta\tau_L - (1-\alpha-\beta)\tau_H + \gamma_T)Y + \\ + \tau_K\delta_K K\big]. \tag{21}$$

From equations (15), (16), (17) and (21), it follows that:

$$I_H = \psi S = \psi A_2\big[(1-\alpha\tau_K - \beta\tau_L - (1-\alpha-\beta)\tau_H + \gamma_T)Y + \\ + \tau_K\delta_K K\big]. \tag{22}$$

$$I_K = (1-\psi)S = (1-\psi)A_2\big[(1-\alpha\tau_K - \beta\tau_L - (1-\alpha-\beta)\tau_H + \\ + \gamma_T)Y + \tau_K\delta_K K\big]. \tag{23}$$

The accumulation of private capital, human capital and public capital is described by the following equations:

$$\dot{K} = I_K - \delta_K K, \quad 0 < \delta_K < 1, \tag{24}$$

$$\dot{H} = G_E + I_H - \delta_H H, \quad 0 < \delta_H < 1. \tag{25}$$

$$\dot{K}_P = G_K - \delta_P K_P, \quad 0 < \delta_P < 1, \tag{26}$$

where δ_i $(i = K, H, P)$ denotes depreciation rates. (Throughout the text, a dot over the symbol for a variable denotes the time derivative, e.g., $\dot{K} = \partial K(t)/\partial t$.) These equations can be transformed to yield the following growth rates:

$$\hat{K} = \frac{\dot{K}}{K} = \frac{I_K}{K} - \delta_K, \tag{27}$$

$$\hat{H} = \frac{\dot{H}}{H} = \frac{G_E + I_H}{H} - \delta_H, \tag{28}$$

$$\hat{K}_P = \frac{\dot{K}_P}{K_P} = \frac{G_K}{K_P} - \delta_P, \tag{29}$$

Substituting (23), equation (27) can be transformed into the following form:

$$\hat{K} = (1-\psi)A_2A_3\frac{Y}{K} + A_4, \tag{30}$$

where

$$A_3 = 1 - \alpha\tau_K - \beta\tau_L - (1-\alpha-\beta)\tau_H + \gamma_T, \tag{31}$$

$$A_4 = \big[(1-\psi)A_2\tau_K - 1\big]\delta_K, \tag{32}$$

Similarly, using (12) and (22) in equation (28) yields:

$$\hat{H} = A_5\frac{Y}{H} + A_6\frac{K}{H} - \delta_H, \tag{33}$$

where

$$A_5 = \gamma_E + \psi A_2 A_3, \tag{34}$$

$$A_6 = \psi A_2 \tau_K \delta_K, \tag{35}$$

Finally, using (3), the growth rates (30) and (33) can be written as:

$$\hat{K} = (1-\psi)A_2A_3A\left(\frac{K}{H}\right)^{\alpha+\beta-1}P^\beta + A_4, \tag{36}$$

$$\hat{H} = A_5A\left(\frac{K}{H}\right)^{\alpha+\beta}P^\beta + A_6\frac{K}{H} - \delta_H. \tag{37}$$

It's worth to compare these 'laws of motion' with equations (36) and (37) in Konopczyński (2014). Note that augmenting the model by adding public capital has significantly complicated the dynamics. Finding the balanced-growth equilibrium in Konopczyński (2014) was relatively simple – equating the right-hand sides of equations (36) and (37) therein and (numerically) solving the resulting nonlinear equation for one unknown (the ratio of K/H). Now, it is more complicated because the 'laws of motion' include an additional variable, P, which evolves over time according to the following equation:

$$\hat{P} = \hat{K}_P - \hat{Y}. \tag{38}$$

Fortunately, the following proposition can be easily proven.

Proposition 1 (proof in the Appendix)

Over time, $P \to \frac{b\gamma_P}{\hat{Y} + \delta_P}$, regardless of whether $\hat{Y}$ is constant or changes over time.

This proposition leads to:

Proposition 2 (proof in the Appendix)

In the long run, the economy converges towards the balanced growth path (hereafter denoted by an overbar), with K, H, K_P and Y growing at the same constant rate (the balanced growth rate, BGR). This balanced growth equilibrium is unique and globally asymptotically stable. The steady-state level of public productive services $\overline{P}$ is related to the BGR through the following formula:

$$\overline{P} = \frac{b\gamma_P}{BGR + \delta_P} = const. > 0 \tag{39}$$

To determine the balanced growth equilibrium analytically, one must solve the system of equations $\hat{Y} = \hat{K} = \hat{H} = \hat{K}_P$. The easiest way to do this is to equate the right-hand sides of equations (36) and (37) and consider equation (39). This consideration results in the following system of two equations with 2 unknowns, K/H and P:

$$(1-\psi)A_2A_3A\left(\frac{K}{H}\right)^{\alpha+\beta-1}P^\beta + A_4 =$$
$$= A_5A\left(\frac{K}{H}\right)^{\alpha+\beta}P^\beta + A_6\frac{K}{H} - \delta_H. \tag{40}$$

$$P = \frac{b\gamma_P}{(1-\psi)A_2A_3A\left(\frac{K}{H}\right)^{\alpha+\beta-1}P^\beta + A_4 + \delta_P}. \tag{41}$$

Having solved this system of equations, one can calculate the BGR by substituting the resulting value of K/H into either (36) or (37).

The system of equations (40) and (41) can only be solved numerically, after substituting certain values for all parameters. Although it is not possible to derive an explicit formula for the BGR, it is possible (and worthwhile) to perform a qualitative sensitivity analysis to determine the relationship between the parameters of the model and the BGR.

2. Qualitative sensitivity analysis

In this section, we investigate how changes in the parameter values influence the BGR. The analysis is performed in three stages. First, we investigate whether an increase in the value of a parameter (e.g., τ_K or γ_P) increases or reduces the values of expressions A_2, …, A_6. Second, using formulas (36) and (37), we investigate whether the graphs of functions $\hat{K}(K/H)$ and $\hat{H}(K/H)$ shift up or down. Third, based on these observations, we conclude whether the intersection of these curves, which corresponds to the BGR (see Appendix, fig. A2), moves up or down.

These stages appear identical to those in the model without public productive services – see section 3 in Konopczyński (2014). However, stage 2 is far more complex than that therein, because functions $\hat{K}(K/H)$ and $\hat{H}(K/H)$ depend on P. This stage must be decomposed into 3 steps. First, we investigate how the aforementioned graphs shift under an artificial assumption that the steady-state value of P is not affected. Second, we investigate how the steady-state value of P changes, and how it shifts the graphs of $\hat{K}(K/H)$ and $\hat{H}(K/H)$. Third, we investigate the combined effects of these two shifts.

As an example of this procedure, we present the analysis of the effects of an increase in the rate of savings γ. It's useful to follow all steps in figure 1, starting from the initial graphs of functions $\hat{K}(K/H)$ and $\hat{H}(K/H)$, labeled $\hat{K}^{old}$ and $\hat{H}^{old}$. The intersection of these curves determines the initial value of BGR, labeled BGR^{old}.

An increase in γ increases the values of A_2, A_4, A_5 and A_6, leaving A_3 unchanged. It follows that the graphs of both functions $\hat{K}(K/H)$ and $\hat{H}(K/H)$ shift up, provided that the steady-state value of P is unchanged. (In figure 1, these new graphs are labeled $\hat{K}^A$ for initial $\overline{P}$; $\hat{H}^A$ for initial $\overline{P}$). If the steady-state value of P remains at its initial level, then the new BGR would increase to the level labeled BGR^A and our analysis would be complete. However, $\overline{P}$ does not remain at its initial level, as a higher BGR implicates a lower $\overline{P}$, in accordance with equation (39). There-

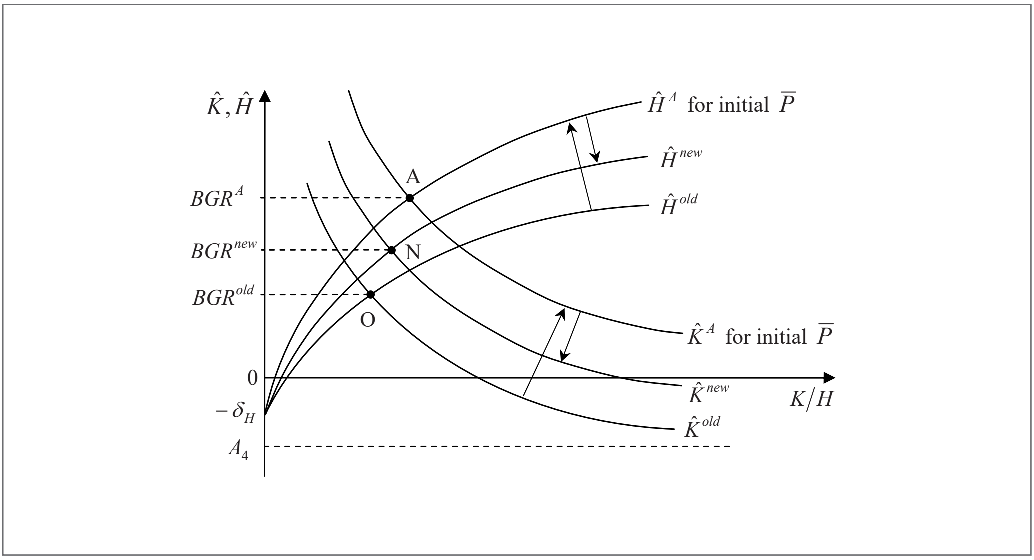

Figure 1. The effects of an increase in the rate of savings γ.

fore, an increase in the BGR reduces $\overline{P}$, which, in accordance with equations (36) and (37) shifts both $\hat{K}(K/H)$ and $\hat{H}(K/H)$ downward to positions labeled $\hat{K}^{new}$ and $\hat{H}^{new}$. This leads to the question: can this downward shift be stronger than the initial upward shift? Can the negative effect of a decrease in $\overline{P}$ entirely offset (or more than offset) the initial positive effect of an increase in the BGR? The answer is: no, it cannot. The next paragraph formally proves this by contradiction.

Let us assume the contrary, i.e., assume that $BGR^{new} \leq BGR^{old}$. Equation (39) implies that $\overline{P}^{new} \geq \overline{P}^{old}$. Recall that we are analyzing the effects of an increase in γ, i.e., $\gamma^{new} > \gamma^{old}$. The last two inequalities imply that $BGR^{new} > BGR^{old}$ because an increase in γ shifts the $\hat{K}(K/H)$ and $\hat{H}(K/H)$ curves up whereas a 'non-reduction' in $\overline{P}$ shifts both curves up or leaves them unchanged. This contradicts our initial assumption. Thus, $BGR^{new} > BGR^{old}$, i.e., an increase in γ unambiguously results in an increase in the BGR.

An increase in the BGR leads to a decrease in $\overline{P}$, i.e., $\overline{P}^{new} < \overline{P}^{old}$. Note that the balanced growth ratio of $\overline{K}/\overline{H}$ will likely change but we do not know the direction of change (it is ambiguous).

The effects of an increase in all other parameters can be traced similarly. In all cases, the initial change (increase/decrease) in the BGR is partially offset by the 'secondary' change (decrease/increase) in $\overline{P}$. The easiest method to prove that this offset is only partial is by contradiction. The results are summarized in table 1.

Notice that increasing any tax rate reduces the BGR and increases $\overline{P}$, with one important exception. The effect of raising the tax rate on capital is ambiguous, as we cannot determine how the $\hat{H}(K/H)$ and $\hat{K}(K/H)$ graphs shift without additional assumptions.

Most of the conclusions are intuitively clear, but some are not. For example, we may have expected a positive relationship between the rate of private savings γ or the rate of public spending on education γ_E and the BGR. To the contrary, the positive relationship between the BGR and the rate of financial transfers to the private sector γ_T requires explanation. Due to the assumption of a permanently balanced government budget, higher transfers to the private sector (with no change in taxes) are automatically offset by reduced public consumption, with no changes in public spending on education or on public capital. These structural changes result in higher disposable income in the pri-

Table 1. Qualitative sensitivity analysis

	$\tau_K \uparrow$	$\tau_H \uparrow$	$\tau_L \uparrow$	$\tau_C \uparrow$	$\gamma \uparrow$	$\gamma_T \uparrow$	$\gamma_E \uparrow$	$\psi \uparrow$	$\gamma_P \uparrow$
A_2	=	=	=	↓	↑	=	=	=	=
A_3	↓	↓	↓	=	=	↑	=	=	=
A_4	↑	=	=	↓	↑	=	=	↓	=
A_5	↓	↓	↓	↓	↑	↑	↑	↑	=
A_6	↑	=	=	↓	↑	=	=	↑	=
graph of $\hat{K}(K/H)$	?	↓	↓	↓	↑	↑	↓	↓	↑
graph of $\hat{H}(K/H)$	?	↓	↓	↓	↑	↑	↑	↑	↑
BGR	?	↓	↓	↓	↑	↑	↑	?	↑
$\overline{P}$	?	↑	↑	↑	↓	↓	↓	?	↑
$\overline{K}/\overline{H}$	?	?	?	?	?	?	↓	↓	?

vate sector. Therefore, private investment in education and physical capital increases whereas government spending on education and public capital remains unchanged. The total effect is unambiguous – the BGR increases and $\overline{P}$ decreases.

The effects of increasing the share parameter ψ are also nontrivial. Recall that ψ represents the share of private savings invested in education. Therefore, increasing ψ increases the rate of human capital accumulation and reduces the rate of physical capital growth. The $\hat{H}(K/H)$ graph shifts up whereas the graph of $\hat{K}(K/H)$ shifts down. Hence, the intersection of these curves unambiguously moves to the left but it is uncertain whether it moves up or down. Again, these shifts are only partially offset by a change (an increase or decrease in $\overline{P}$), which can be proven by contradiction. A higher ψ unambiguously reduces the balanced growth ratio of $\overline{K}/\overline{H}$ – there is more human capital per unit of physical capital. However, the relationship between ψ and the BGR is ambiguous.

The effects of an increase in γ_P, summarized in table 1, require more detailed explanation. Note that an increase in γ_P has no influence on A_2, A_3, A_4, A_5 or A_6. However, an increase in γ_P directly translates into an increase in $\overline{P}$ – see equation (39). Assume that BGR is at its initial level. An increase in $\overline{P}$ shifts the graphs of $\hat{K}(K/H)$ and $\hat{H}(K/H)$ up. (In figure 2, these new graphs are labeled $\hat{K}^A$ for the initial BGR and $\hat{H}^A$ for the initial BGR, respectively). The BGR does not remain at its initial level – it increases. A higher BGR implies a lower $\overline{P}$, in accordance with equation (39). In accordance with equations (36) and (37), this shifts $\hat{K}(K/H)$ and $\hat{H}(K/H)$ down to the positions labeled $\hat{K}^{new}$ and $\hat{H}^{new}$. This shift leads to the question: can this downward shift be stronger than the initial upward shift? Can the (secondary) decrease in the BGR entirely offset (or more than offset) the initial increase in the BGR? The answer is: no, it cannot. Let us prove this by contradiction.

Let us assume the contrary, i.e., assume that $BGR^{new} \le BGR^{old}$. This implies that $\overline{P}^{new} \le \overline{P}^{old}$, as the $\hat{K}(K/H)$ and $\hat{H}(K/H)$ curves shift due to changes in $\overline{P}$. Recall that we are analyzing the effects of an increase in γ_P, i.e., $\gamma_P^{new} > \gamma_P^{old}$. Note that these three inequalities contradict each other, which follows from equation (39). Hence, our initial assumption is false. Thus, $BGR^{new} > BGR^{old}$, i.e., an increase in γ_P unambiguously results in an increase in the BGR. It also fol-

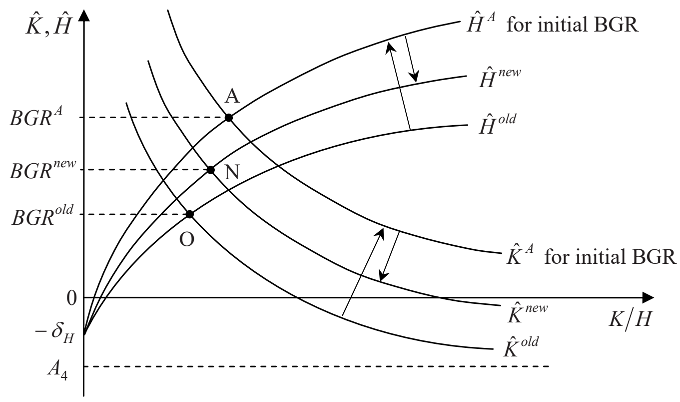

Figure 2. The effects of an increase in γ_P.

lows that an increase in γ_P results in an increase in $\overline{P}$. (Formally, $\overline{P}^{new} > \overline{P}^{old}$). Finally, note that the change in the ratio of $\overline{K}/\overline{H}$ is ambiguous.

Based on table 1, we can formulate the following.

Proposition 3
First, the BGR is an increasing function of the rate of private savings γ, the rate of public transfers γ_T, the rate of public spending on education γ_E, and the rate of government spending on public capital γ_P. Second, the BGR is a decreasing function of the tax rates on labor, human capital and consumption. Third, the relationship between the BGR and the tax rate on capital income and the share coefficient (the percentage of private savings invested in education) are ambiguous.

Although interesting, these *qualitative* results only enhance our desire for *quantitative* results. Moreover, as the BGR cannot be determined analytically, it is not possible to determine *how strongly* changes in the parameters influence the BGR. We know the *direction* of the effect, but we do not know the *size* of the effect. Answering these questions requires calibrating the model and performing numerical analyses. We calibrate the model for Poland and numerically analyze the optimal fiscal policy and private sector parameters. The calibration is based on macroeconomic data for Poland for 2000 – 2015, published by the Eurostat, IMF, OECD, and the Kiel Institute for the World Economy.

3. Model calibration for Poland

Initial stock of private and public capital

Statisticians have difficulty obtaining reliable data on the stock of public capital, even for OECD countries. For example, consider two large, reliable databases constructed by the IMF (IMF 2015) and the KIEL Institute (Kamps 2006). For the majority of OECD countries, the time series reported by both institutions diverges significantly, even for countries such as the United States and Germany (see fig. 3) Moreover, there are large discrepancies although both institutions applied similar definitions of public capital and the same methodology (the Perpetual Inventory Method), started their calculations at the same base year (1860), and applied similar assumptions regarding depreciation patterns over time (in fact, the IMF borrows these assumptions from the KIEL Institute).

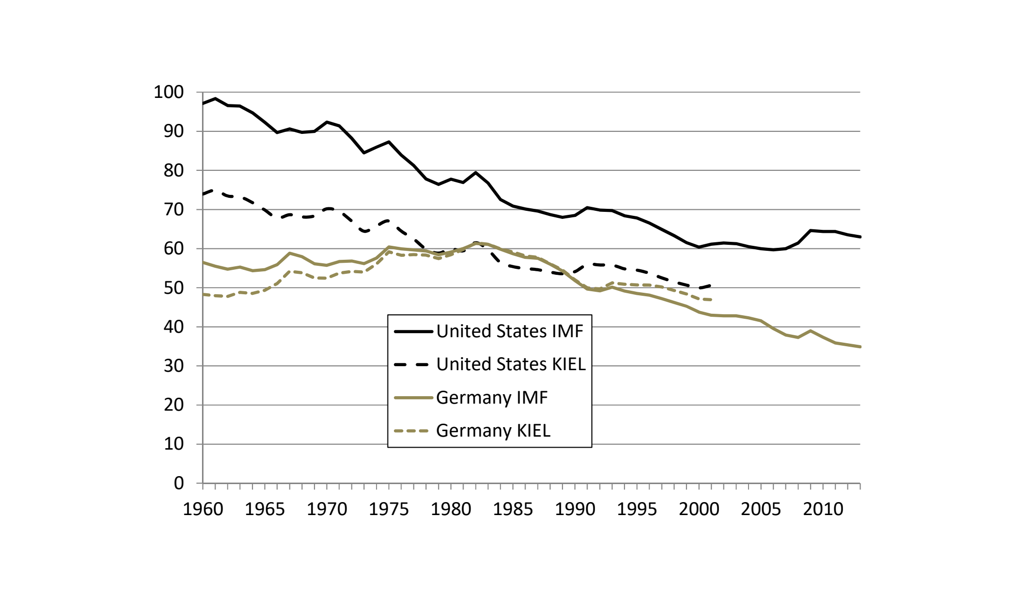

Figure 3. Public capital as a percentage of the GDP.
Source: Adapted from "Making public investment more efficient" by International Monetary Fund (2015). Retrieved from http://www.imf.org/external/np/pp/eng/2015/061115.pdf; "New Estimates of Government Net Capital Stocks for 22 OECD Countries, 1960– 2001" by C. Kamps 2006, IMF Staff Papers, 53(1), 120–50.

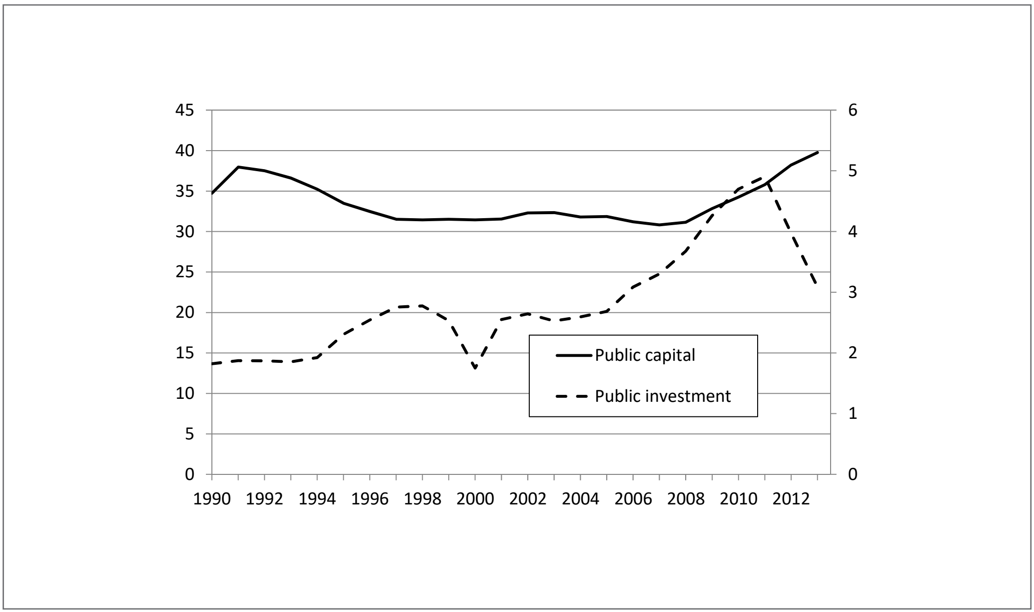

Figure 4. Public capital (left-hand scale) and public investment (right-hand scale) in Poland as a percentage of the GDP.
Source: "IMF Investment and Capital Stock Dataset", by International Monetary Fund (2016). Available at http://www.imf.org/external/np/fad/publicinvestment/

In addition, there are also large differences between countries. For example, according to KIEL, in 2001 (the last year in their database) the stock of public capital (as % of the GDP) varied from less than 40% (in Ireland, Belgium, Canada, UK) to 75% in New Zealand and 120% in Japan. IMF provides statistics for a much wider group of countries, so the differences between countries are even larger (and they persist over time). For example, in 2013, according to IMF, public capital was approximately 35% of the GDP in Argentina, Brazil and Germany and 107% in Japan, 137% in China, and 147% in Malaysia.

Apart from these discrepancies, the databases provided by IMF and KIEL Institute indicate that the ratio of public capital to GDP is not necessarily increasing over time in most OECD countries (as one may expect). In most countries, it has been stable or decreasing in the last 2 decades. This is true for most developed countries, which may indicate that public capital across the world is in decline (perhaps underinvested, as suggested by Dobbs et al. 2013) or is steadily losing importance for economic growth in the richest countries (which is an intriguing empirical question) or both. Let us leave this issue for future research, and turn to Poland.

KIEL does not provide data for Poland but IMF does. Figure 4 presents the overall government investment (gross fixed capital formation) and stock of public capital as a percentage of the GDP.

The evolution of public capital in Poland can be divided into three general phases. During the first phase, which started immediately after political and economic revolution (1989), the stock of public capital deteriorated from approximately 35% of the GDP in 1990 to 31.5% in 1997 due to low investment in public infrastructure, 2.1% of the GDP on average (a sharp increase in 1991 is a statistical effect of the deep recession in 1991 when the GDP decreased by 7% rather than an increase in public capital). In the following decade (1998–2008), the level of public capital remained stable at approximately 31.5% of the GDP. Over the third phase, which started in 2008, Poland enjoyed a significant increase in public investment (mainly due to large EU convergence funds), resulting in unprecedented improvement in public capital that reached 40% of the GDP in 2013.

For our simulations, we assume that the initial stock of public capital is 33% of the GDP and the level of public investment: $\gamma_P = 3.3\%$ (the arithmetic averages for 2000–2013).

Technological parameters

The elasticities of the production function (1) have been estimated in many papers, e.g., Mankiw, Romer, Weil (1992), Manuelli and Seshadri (2005). Studies focusing on Poland include Cichy (2008) and Próchniak (2013). The estimated values are typically close to 1/3; hence, we set: $\alpha = \beta = 1 - \alpha - \beta = 1/3$. As we argued in Konopczyński (2014), the rate of physical capital depreciation is difficult to estimate due to rapid economic transformation that resulted in a large amount of obsolete machinery and infrastructure inherited from the centrally 'planned' economy. In various research papers regarding OECD countries, physical capital depreciation varies from approximately 3.5% to 7%, and we assume that $\delta_K = 5.5\%$. The rate of human capital depreciation has been estimated by Manuelli and Seshadri (2005), Arrazola and de Hevia (2004) and others. Following these authors, we set $\delta_H = 1.5\%$.

There are no statistics regarding the rate of depreciation of public capital in Poland. Therefore, we follow methodology applied by the IMF (2015), which follows assumptions adopted by Gupta et al. (2014). They argue that "country-specific depreciation rates (…) are likely to increase with income assuming that the share of assets with a shorter life spans (such as technology assets) rises with income levels". They assume that public capital in middle income countries depreciates at a rate of 3.51% per year and 4.59% in high income countries. Poland is located somewhere between these two groups, so we set $\delta_P = 4\%$.

Next, we assess the real rate of return on capital (r). From (5), it follows that $r = \alpha \cdot Y/K - \delta_K$. The ratio of Y/K is difficult to estimate for Poland – we exposed major problems in Konopczyński (2014), section 4. The available data for Poland only reflect a share of the productive capital – namely, the "gross value of fixed assets". Therefore, in Konopczyński (2014) we applied the average ratio from the Kiel database, i.e., we set $Y/K = 1/3$. However, we need to alter this number because our model separates public and private capital. Subtracting public capital (33% of the GDP) yields: $Y/K = 1/(3 - K_P/Y) = 1/(3 - 0.3\;\;) = 0.375$. Substituting this value into (5) yields the real rate of return on private capital, $r = 6.98\%$. This outcome is very close to most long-term empirical estimates for OECD countries. For example, Campbell, Diamond and Shoven (2001) reported that the average real rate of return on stocks in the U.S. from 1900–1995 was 7%. In our opinion, analogous indicators for the Polish stock market are irrelevant because the Polish stock market is still young and volatile, and thus does not reflect the long-term equilibrium.

Social transfers and the rates of savings and investment

Cash transfers to the private sector (pensions, various benefits, social assistance, etc.) were 15.1% of the GDP from 2000-2015. Thus, we set $\gamma_T = 15.1\%$.

The average rate of savings can be calibrated based on equation 15), which can be transformed into the following formula:

$$\gamma = \frac{S}{Y_d} = \frac{I_K + I_H}{Y - T + G_T} = \frac{I_K/Y + I_H/Y}{1 - T/Y + G_T/Y}. \tag{42}$$

According to Eurostat, the average gross fixed capital formation in Poland from 2000-2015 was 20.3% of the GDP. The average private spending on education from 2000-2011 (the latest data available from Eurostat) was 0.65% of the GDP. The ratio of 'total receipts from taxes and social contributions' to the GDP from 2000-2015 was 33.4% (and very stable). Substituting these numbers into (42) yields $\gamma = 25.64\%$.

The share parameter ψ can be calculated directly from equation (17): $\psi = \frac{I_H}{S} = \frac{I_H/Y}{I_K/Y + I_H/Y} = \frac{0.65\%}{20.3\% + 0.65\%} = 3.10\%$. In Poland, a mere 3.1% of private savings is invested in education. However, private spending on education is probably underestimated in official statistics – Eurostat only considers "school fees; materials such as textbooks and teaching equipment; transport to school (if organized by the school); meals (if provided by the school); boarding fees; and expenditure by employers on initial vocational training". All other private expenses related to education are classified as consumption, e.g., the cost of accommodation, travel, books, etc.

The average public expenditure on education in Poland from 2000-2011 (the latest available data) was 5.20% of the GDP (Eurostat); hence, based on formula (12), we set $\gamma_E = 5.20\%$.

Average tax rates

From 2000-2015, consumption taxes were 12.0% of the GDP. Thus, the ratio of income taxes to the GDP was $T_1/Y = T/Y - T_2/Y = 33.4\% - 12.0\% = 21.4\%$. Eurostat reports 'implicit tax rates' on capital, labor and consumption. In Poland, from 2000-2012 (the latest available data), the average rates were: $\tau_K = 20.5\%$, $\tau_L = 32.8\%$, and $\tau_C = 19.5\%$, respectively. Note that the implicit tax rate on labor is defined as the "*Ratio of taxes and social security contributions on employed labor income to total compensation of employees*". To the best of our knowledge, there are no data on the average tax rates on human capital. As noted in Konopczyński (2014), some researchers suggest that, in countries with highly progressive taxes on personal income, tax rates on human capital must be higher than the tax rates on (raw) labor. However, in Poland, the size of the tax wedge on labor is nearly independent of the level of income, i.e., the effective tax rates on wages are nearly linear. Thus, it is reasonable to assume that the average tax rates on human capital and raw labor in Poland are identical, i.e., $\tau_H = \tau_L$.

Recall that, according to Eurostat, $\tau_L = 32.8\%$. However, if we set $\tau_H = \tau_L = 32.8\%$ and perform the calibration, the model significantly overestimates the total revenue from income taxes (by approximately 7% of the GDP). This problem arises because our concepts of human capital and raw labor are wider than the definitions employed by Eurostat. Eurostat classifies "taxes on income and social contributions of the self-employed" as part of the capital income tax – a detailed explanation can be found in the methodological publication by Eurostat (2010), Annex B. However, self-employed entrepreneurs correspond to our concept of human capital (as well as a part of raw labor). Self-employment is very popular in Poland – there are millions of small, family businesses and many individuals operate single-person firms and provide services for larger enterprises. Note that the tax rate on capital income published by Eurostat is much lower (20.5%) than the tax rate on labor (32.8%). Therefore, in our model, the tax rates on human capital and labor should be somewhere between these two numbers. As there are no additional statistics, we calibrated both rates at this level, at which the model produces a total share of taxes of the GDP that is consistent with statistics (33.4%, see above). Thus, we obtain $\tau_H = \tau_L = 26.6\%$, i.e., rates that are approximately 20% lower than those reported by Eurostat.

The next step in the calibration is computing the expressions A_i. We do not report these values here as they do not have any economic interpretation. Knowing these values and using formula (30), we compute the average capital growth rate from 2000-2015: $\hat{K} = (1-\psi)A_2 A_3 Y/K + A_4 = 2.10\%$. Substituting (13) into (29) yields $\hat{K}_P = \frac{\gamma_P}{K_P/Y} - \delta_P$. Substituting the statistical values leads to the following: $\hat{K}_P = \frac{3.3\%}{33\%} - 4\% = 6.0\%$. This implies that the flow of public services was increasing at the following rate: $\hat{P} = \hat{K}_P - \hat{Y} = 6.0\% - 3.66\% = 2.34\%$, which is approximately the private capital growth rate.

From equation (3), it follows that:

$$\hat{Y} = (\alpha+\beta)\hat{K} + (1-\alpha-\beta)\hat{H} + \beta\hat{P}, \tag{48}$$

The average GDP growth rate in Poland from 2000-2015 was 3.66% (geometric mean). We can estimate the human capital growth rate based on the basis of equation (48), from which it follows that $\hat{H} = \frac{\hat{Y} - (\alpha+\beta)\hat{K} - \beta\hat{P}}{(1-\alpha-\beta)} = \frac{3.66\% - 2/3 \cdot 2.1\% - 1/3 \cdot 2.34\%}{1/3} =$ $= 4.43\%$.

These results imply that, from 2000-2015, economic growth in Poland was primarily driven by rapid growth in the stock of human capital coupled with accumulation of public capital (however, the latter was only after 2005, see fig. 1), and secondarily by the accumulation of (private) productive capital. An impressive increase in human capital in Poland is a well-known 'stylized fact', confirmed by a sharp increase in the number of students, PhDs, etc.

For simulations, it is necessary to estimate parameter A. First, from equation (33), we calculate the proportion $K/H = \frac{\hat{H} + \delta_H}{A_5 Y/K + A_6} = 2.7083$. Transforming formula (3) yields

$$A = \frac{Y}{K^{\alpha+\beta} H^{1-\alpha-\beta} P^{\beta}} = \frac{Y}{K}\left(\frac{K}{H}\right)^{1-\alpha-\beta} \frac{1}{P^{\beta}}, \tag{51}$$

which, using (2), can be written as

$$A = \frac{Y}{K}\left(\frac{K}{H}\right)^{1-\alpha-\beta}\left(\frac{K_P}{Y}\right)^{-\beta}, \tag{52}$$

Substituting $Y/K = 1/3$, $K/H = 2.7083$, and $K_P/Y = 0.33$ yields $A = 0.7555$.

To perform the simulations, we assume initial values of the variables K, H, L and K_P. Two of these (K and L) can be determined freely, provided that we confine our interest to the rates of growth and relationships (the proportions) among variables. Therefore, we set $L(0) = 1$ and $K(0) = 267$. This choice is convenient, as the initial GDP is 100, so the initial values of all the other variables are identical to their percentage shares of the GDP. Given $K/H = 2.7083$ and $K_P/Y = 0.33$, it follows that $H(0) = 98.58$ and $K_P(0) = 33$.

In summary, we have the following base set of parameters and endowments:

$$A = 0.7555,\ \alpha = 1/3,\ \beta = 1/3,\ \delta_K = 5.5\%,\ \delta_H = 1.5\%,\ \gamma = 25.64\%,\ \psi = 3.10\%,\ \gamma_E = 5.20\%,\ \gamma_T = 15.1\%,\ \tau_K = 20.5\%,\ \tau_C = 19.5\%,\ \tau_H = \tau_L = 26.6\%,\ L(0) = 1,\ K(0) = 267,\ H(0) = 98.58,\ K_P(0) = 33. \tag{53}$$

4. Baseline scenario

The baseline scenario with the set of parameters (53) reproduces actual statistics on the Polish economy from 2000-2015 and reproduces the factual (average) ratios of the following variables to the GDP: C, I_K, I_H, T_1, T_2, G_T, G_E, G_K as well as the (average) rate of GDP growth. The rates of growth for $t = 0$ generated by the model in the baseline scenario are

$$\hat{Y} = 3.66\%,\ \hat{K} = 2.10\%,\ \hat{H} = 4.43\%,\ \hat{K}_P = 6.0\%.$$

These rates are not equal; hence, the Polish economy is not yet on a balanced growth path. Using the procedure described at the end of section 2, we can numerically obtain the BGR in the baseline scenario, which is 3.70% – slightly higher than the average growth rate from 2000-2015. The process of convergence towards a balanced growth path is presented in figure 5, which illustrates the trajectories of several growth rates in the baseline scenario.

Having augmented the model with public capital, we now update our previous simulations (published in Konopczyński 2014) regarding tax rates and (private and public) spending on education. More importantly, however, the augmented model allows for simulating the effects of changes in the level of public investment into public capital.

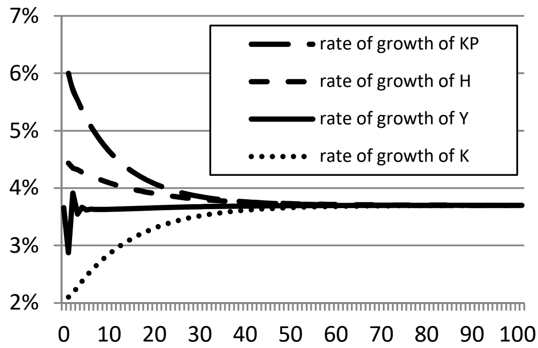

Figure 5. Convergence to a balanced growth path in the baseline scenario.

5. Selected tax-cut scenarios in Poland

As in Konopczyński (2014), let us consider two types of scenarios:

a) reducing a given tax rate by 1 or 5 percentage points (pp),

b) reducing all tax rates by 1 or 5 pp.

Table 2 contains the BGRs calculated under all these scenarios. In all cases, the economy grows faster (on the balanced growth path) than in the baseline scenario. To visualize the long-term (welfare) effects, we also include numbers indicating by how many percent GDP exceeds the baseline GDP after 30 years (in table 2, numbers in bold). These indicators are calculated as follows:

$$gain\ \ after\ 30\ years = $$

$$= \frac{Y(t=30)\ in\ selected\ scenario}{Y(t=30)\ in\ the\ baseline\ scenario} - 1\,. \qquad (54)$$

In each scenario, the tax rates are reduced at $t = 0$.

Unsurprisingly, the most favorable results are associated with the largest tax cuts, i.e., the scenario of reducing all tax rates by 5 pp. After 30 years, the GDP would be 10.2% higher than that under the baseline scenario. Table 3 shows some structural changes resulting from such a reduction of taxes.

Cutting all tax rates by 5 pp would reduce the overall tax burden from the recent 33.4% to 27.1% of the GDP, which would bring Poland much closer to the levels observed in the United States (approx. 25%), South Korea (26%) and Japan (27%). The immediate effect of the reduction in taxes would be an increase in private sector savings relative to the GDP (from 20.8% to 22.6%), an increase in investment (from 20.2% to 21.9% of the GDP), and an increase in private expenditures on education. The accelerated accumulation of both physical and human capital would shift the economy towards a higher balanced growth path. As a result, the BGR would increase by approximately 0.32 percentage points.

Notably, this scenario is associated with significant structural changes in the economy. Reduced tax receipts while maintaining 15.1% of the GDP for cash social transfers (primarily pensions) and 5.2% of the GDP for public expenditures on education would negatively affect public consumption expenditures. This gap would have to be (partially) offset by

Table 2. Simulation results for Poland - different tax cut scenarios

	1 pp reduction	5 pp reduction
τ_L	3.71%	3.77%
	0.4%	**2.2%**
τ_K	3.71%	3.74%
	0.3%	**1.3%**
τ_H	3.71%	3.77%
	0.4%	**2.2%**
τ_C	3.73%	3.83%
	0.8%	**4.1%**
reduction of all tax rates simultaneously	3.76%	4.02%
	1.9%	**10.2%**

Table 3. The scenario of simultaneously reducing all tax rates by 5 pp.

the BGR and structural indicators (%)	baseline scenario	reduction of all tax rates by 5 pp
the BGR	3.70	4.02 (the effect after 30 years= +10.2%)
C/Y	60.4	65.4
T/Y	33.9	27.1
S/Y	20.8	22.6
I_K/Y	20.2	21.9
G_P/Y	3.3	3.3
G_E/Y	5.2	5.2
I_H/Y	0.65	0.70
K/Y	2.19	2.30
K_P/Y	0.43	0.41

increased consumption spending in the private sector. As a result of the tax cuts, this would occur naturally. Under the scenario presented in table 3, the share of private consumption of the GDP increases from 60.4% to 65.4%. This would bring the Polish economy structurally closer to that of the United States, where private consumption is approximately 70% of the GDP.

6. Selected scenarios of increasing public and private spending on education

This section contains an update of three scenarios discussed in section 8 of Konopczyński (2014):

A) the government increases public spending on education by 1 pp of the GDP at the expense of public consumption.

Table 4. Scenarios of increasing public and private spending on education

The BGR and structural indicators (%)	Baseline scenario $\gamma_E = 5.20\%$ $\gamma = 25.64\%$ $\psi = 3.10\%$	A Increase in public spending on education by 1 pp of GDP $\gamma_E = 6.20\%$	B Increase in private savings by 1 pp of GDP $\gamma = 26.80\%$ $\psi = 3{,}10\%$	C Increase in private spending on education by 1 pp of GDP $\gamma = 26.84\%$ $\psi = 7.54\%$
the BGR	3.70	4.00 GDP effect after 30 years **+8.2%**	3.89 GDP effect after 30 years **+5.8%**	4.00 GDP effect after 30 years **+8.3%**
C/Y	60.4	60.3	59.6	59.5
T/Y	33.9	33.9	33.7	33.8
S/Y	20.83	20.81	21.83	21.83
I_K/Y	20.18	20.17	21.15	20.18
G_P/Y	3.3	3.3	3.3	3.3
G_E/Y	5.2	6.2	5.2	5.2
I_H/Y	0.65	0.65	0.68	1.65
K/Y	2.19	2.12	2.25	2.13
K_P/Y	0.43	0.41	0.42	0.41

B) private sector savings increase by 1 pp of the GDP (at the expense of individual consumption), with an unchanged structure of investment expenditures (i.e., an unchanged ψ). As a result, private investment in physical and human capital increase by a total of 1 pp of the GDP.

C) private sector savings increase by 1 pp of the GDP (at the expense of individual consumption) and additional savings are spent solely on education (for this purpose, ψ has been appropriately amended). Private spending on education increases by 1 pp of the GDP at the expense of private consumption.

Table 4 presents the results.

The conclusions are similar to the results obtained in Konopczyński (2014). With respect to the BGR, all three scenarios significantly outperform the baseline scenario. However, the effect of additional spending on education (scenarios A and C) is stronger than the effect of a similar increase in private savings, with additional resources primarily spent on investments in physical capital (97%). It is clearly much better to spend the additional money on education rather than on physical capital. Moreover, comparing scenarios A and C shows that it is relatively unimportant whether the additional funds for education come from a reduction in public or private consumption.

7. The optimal structure of private investment

Investing in human capital (education) is of crucial importance for economic growth. However, as shown in section 2, the relationship between the BGR and the share parameter ψ (the share of private savings spent on education) cannot be established analytically. Therefore, using the baseline scenario as a benchmark, we calculated the BGR corresponding to a range of ψ. Figure 6 pres-

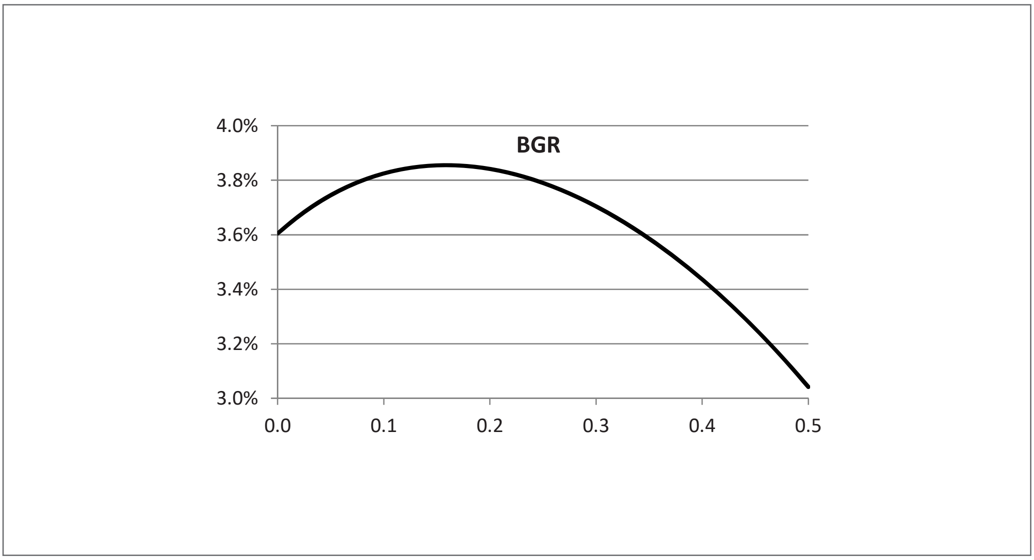

Figure 6. The BGR as a function of the share parameter ψ .

ents the results. The BGR reaches a maximum (3.855%) at $\psi = 16\%$. The current structure of private investment in Poland is far from optimal: households should spend 16% of their savings on education, rather than a mere 3.1%, as Eurostat reports. However, that private spending on education is underestimated in official statistics – a substantial share of it is classified as consumption (e.g., the cost of accommodation, travel, books, etc.).

8. Increasing investment in public capital

Table 5 presents the results of two scenarios:

D) increasing spending on public capital by 1 pp of the GDP at the expense of public consumption.

E) increasing spending on public capital by 1 pp of the GDP at the expense of public spending on education.

These scenarios yield some remarkable conclusions. First, if we assume that increasing expenditures on public capital requires a reduction in public spending elsewhere, these scenarios prove that it's better to cut public consumption rather than public spending on education.

Second, both scenarios significantly outperform the baseline scenario, so the stock of public capital in Poland is still too low. Note that, in both scenarios, the stock of public capital increases from the current level of 40% of the GDP (in 2013) to 52% in scenario D and 55% in scenario E. These scenarios would bring the Polish economy structurally closer to more developed countries, where usually this indicator is 40 to 60%, e.g., according to the IMF, in 2013 it was 63% in the United States, 60% in Italy, 51% in Canada and France, and a mere 35% in Germany. Notably, it was an outstanding 107% in Japan.

Third, it follows from scenario D that it's worth increasing investment in public capital at the cost of public consumption. This proposition is intuitive and does not require deeper analysis; however, it could be politically difficult. In contrast, scenario E implies something far less obvious (and politically controversial): it's worth transferring *certain parts of* public resources in Poland from education to public capital. As a result, the BGR increases significantly. Scenario E assumes that the size of this transfer is 1% of the GDP. We also generalized this scenario and investigated the results of different "shift parameters", from 0% to 2.5% of the GDP. Figure 7 presents the results. The BGR reaches a maximum (3.85%), if the Polish government shifts approx. 1.3% of the GDP from education to public investment. After 30 years, the GDP would be 8.1% higher than that under the baseline scenario.

Table 5. Scenarios of increasing investment in public capital

The BGR and structural indicators (%)	Baseline scenario $\gamma_P = 3.3\%$ $\gamma_E = 5.20\%$	D Shift from public consumption to public capital $\gamma_P = 4.30\%$ $\gamma_E = 5.20\%$	E Shift from spending on education to spending on public capital $\gamma_P = 4.3\%$ $\gamma_E = 4.20\%$
the BGR	3.70	4.20 GDP effect after 30 years **+16.9%**	3.84 GDP effect after 30 years **+6.8%**
C/Y	60.4	60.3	60.4
T/Y	33.9	34.0	33.9
S/Y	20.83	20.80	20.82
I_K/Y	20.18	20.15	20.17
G_P/Y	3.3	4.3	4.3
G_E/Y	5.2	5.2	4.2
I_H/Y	0.65	0.65	0.65
K/Y	2.19	2.08	2.16
K_P/Y	0.43	0.52	0.55

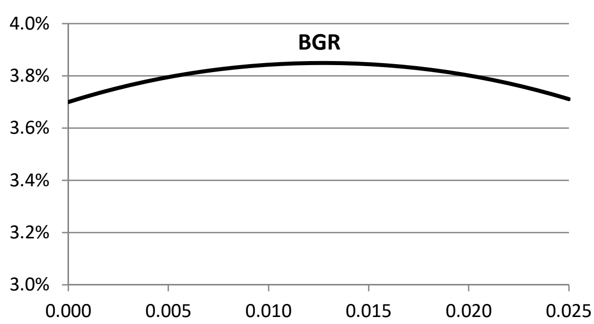

Figure 7. The BGR as a function of the "shift parameter" from public spending on education to public capital.

Table 6. Changes in tax rates accommodated by public capital or public education

The BGR and structural indicators (%)	Baseline scenario $\gamma_E = 5.20\%$ $\gamma_P = 3.3\%$	F The reduction of all tax rates by 1 pp at the cost of public capital $\gamma_E = 5.20\%$ $\gamma_P = 1.75\%$	G The reduction of all tax rates by 1 pp at the cost of public education $\gamma_E = 3.77\%$ $\gamma_P = 3.3\%$	H The increase in all tax rates by 1 pp to the benefit of public capital $\gamma_E = 5.20\%$ $\gamma_P = 4.79\%$	I The increase in all tax rates by 1 pp to the benefit of public education $\gamma_E = 6.61\%$ $\gamma_P = 3.3\%$
the BGR	3.70	2.68 GDP effect after 30 years **–26.0%**	3.26 GDP effect after 30 years **–10.1%**	4.35 GDP effect after 30 years **+22.5%**	4.04 GDP effect after 30 years **+9.3%**
C/Y	60.4	61.6	61.5	59.3	59.4
T/Y	33.9	32.3	32.4	35.3	35.3
S/Y	20.8	21.2	21.2	20.4	20.5
I_K/Y	20.2	20.6	20.5	19.8	19.8
G_P/Y	3.3	1.7	3.3	4.8	3.3
G_E/Y	5.2	5.2	3.8	5.2	6.6
I_H/Y	0.65	0.66	0.66	0.63	0.64
K/Y	2.19	2.52	2.34	2.01	2.08
H/Y	1.12	1.40	0.93	1.00	1.31
K_P/Y	0.43	0.26	0.45	0.57	0.41

9. Changes in tax rates accommodated by public capital or public education

All tax-cut scenarios presented in section 5 assume that lower tax revenue is accommodated by an appropriate reduction in public consumption. As public consumption does not have any productive effect in our model, it is evident that such tax cuts must result in a higher long-term growth rate (the BGR). In this section, we study less obvious scenarios in which reduced tax revenue does not reduce public consumption, rather, other components of public spending are reduced. In all scenarios in this section, we assume that the level of public consumption (as a share of the GDP) is at the baseline steady-state level of 10.3% of the GDP and the government budget is always balanced. Any changes in tax rates are accommodated by an appropriate adjustment in public investment, or public expenditures on education. Table 6 presents the results of four scenarios:

F) simultaneous reduction of all tax rates by 1 pp at the cost of public capital,
G) simultaneous reduction of all tax rates by 1 pp at the cost of public education,
H) simultaneous increase of all tax rates by 1 pp to the benefit of public capital,
I) simultaneous increase of all tax rates by 1 pp to the benefit of public education.

Scenarios F and G clearly show that a reduction in tax rates at the cost of government investment in public capital or education is a very bad idea. In both cases, the BGR drops significantly – due to a huge deterioration in public capital in scenario F (the long-term ratio of public capital to the GDP decreases from 43%

to 26%) and due to significant deterioration in human capital in scenario G (the long-term ratio of human capital to the GDP decreases from 1.12 to 0.93). Notably, although both scenarios F and G are very bad, G is the lesser of the two evils. Thus, if the government is forced to cut public expenditures, it should cut public consumption to benefit the economy – see section 5. If the government is forced to cut expenditures on public capital or education, it should choose the latter as the lesser evil. Although controversial, this conclusion is consistent with scenario E – see section 8 above.

Scenarios H and I are the reverse of F and G, respectively. Rather than cutting taxes, the government raises all tax rates by 1 percentage point in both scenarios and spends the extra revenue on public capital (scenario H) or education (scenario I). Both scenarios outperform the baseline scenario and it's better to invest the extra revenue into public capital rather than education. This conclusion is evidently controversial but is confirmed by scenario E.

10. Robustness of the results

As mentioned in section 7, private spending on education in Poland is likely underestimated in official statistics – a substantial share of it is classified as private consumption. In our calibration, we assumed that the official statistic on private spending on education is accurate, i.e., a mere 0.65% of the GDP. One might suspect that if this ratio was higher, the results and conclusions of this paper might be different. Fortunately, this is not the case. All our simulations (scenarios) are very robust to a change in the level of private spending on education. We verified this by assuming that the private sector spends twice as much on education, i.e., 1.3% of the GDP, and recalibrating the model and re-calculating the scenarios presented above. The baseline scenario and other scenarios were nearly unaffected. For example, a reduction in all tax rates by 5 pp. (table 2) would increase the BGR to 4.03% instead of 4.02%; in scenarios A and C, the BGR would be 3.96% instead of 4.00%; scenarios B and D would be completely unaffected; in scenario E, the BGR would be 3.88% instead of 3.84%; and in scenario F, the BGR would be 2.51% instead of 2.65%.

Summary

We have demonstrated that the economy converges towards a balanced growth equilibrium that is globally asymptotically stable. Despite the simplicity of the model, the balanced growth rate (BGR) can only be calculated numerically, as it requires solving a complex system of two non-linear equations.

The BGR is an increasing function of the rates of private savings, public transfers, public spending on education, and government spending on public capital. The BGR is also a decreasing function of tax rates on labor, human capital and consumption. The relationship between the BGR and the tax rate on capital income and the share coefficient (the percentage of private savings invested in education) are ambiguous. As in Konopczyński (2014), this ambiguity is a property of the theoretical model and implies that these relationships are dependent on specific parameter values. Thus, the relationship between the tax rate on capital and the BGR can be positive or negative depending on the parameter values. The relationship between the tax rate on capital income and the BGR may be positive or negative, depending on a particular set of values of parameters.

Our calibration of the model for Poland leads to several empirical conclusions, which can be summarized as follows. From 2000-2015, economic growth in Poland was primarily driven by a rapid increase in the stock of human capital (at 4.4% per annum) coupled with fast accumulation of public capital (approximately 6% per annum), and secondarily by the accumulation of private capital (2.1% annually). The baseline scenario suggests that Poland will converge to a balanced growth path with the GDP growing at the BGR of 3.7%. However, this rate depends on the long-term (average) values of certain instruments of fiscal policy. For example, reducing income and consumption tax rates by 5 percentage points would increase annual GDP growth by approximately 0.32 percentage points, which would result in a cumulative 10% increase in the GDP after 30 years relative to the baseline scenario (assuming current tax rates). Such a reduction in all tax rates would bring the Polish economy structurally closer to countries such as the United States or South Korea with significantly lower tax burdens (25-27% of the GDP compared to 33% in Poland) and much higher share of individual consumption (approximately 70% of the GDP compared to 60.4% in Poland).

Investing in human capital (education) is essential for economic growth. The conclusions are similar

 DOI: 10.5709/ce.1897-9254.281

(though not identical) to the results obtained in the model without public capital. An increase in education expenditures by 1 percentage point of the GDP would have a similar long-term effect as simultaneously reducing all tax rates by 5 percentage points. The GDP growth rate would increase by approximately 0.3 percentage points. Whether the additional funding for education comes from a reduction in public or private consumption is irrelevant.

In Poland only 3% of private savings is currently spent on education. We show that, to maximize the BGR, it should be as high as 16%. Therefore, the current structure of private investment in Poland is far from the optimum. However, that private spending on education is likely underestimated in official statistics – a substantial share of it is classified as consumption.

Finally, despite a significant increase in the level of public capital over the last decade, it is still too low. Increasing spending on public capital by 1 pp of the GDP at the cost of public consumption would increase the BGR by 0.5 percentage points. Remarkably, even diverting resources from education to public infrastructure would slightly increase the BGR (by 0.14 percentage points).

Despite the relative methodological simplicity, our analysis provides qualitative and quantitative insights into the positive effects of investing in education and public capital on economic growth in Poland as well as the negative consequences of taxes. Our model captures certain 'stylized facts', especially the rapid accumulation of human capital over the past 2 decades coupled with an equally fast accumulation of public capital over the last decade. However, our model neglects certain phenomena that influence the Polish economy. For example, Poland experienced large inflows of FDI and portfolio investment. Considerable migration from Poland to other EU countries also occurred. These two phenomena are not included in our model and may presumably offset one another to some extent.

References

Agénor, P. R., & Yilmaz, D. V. (2011). The tyranny of rules: fiscal discipline, productive spending, and growth in a perfect foresight model. *Journal of Economic Policy Reform*, 14(1), 69-99. https://doi.org/10.1080/17487870.2010.503086

Arrazola, M., & de Hevia, J. (2004). More on the estimation of the human capital depreciation rate. *Applied Economic Letters*, 11(3), 145-148. https://doi.org/10.1080/1350485042000203742

Arrow, K. J. & Kurz, M. (1970). Public Investment, the Rate of Return and Optimal Fiscal Policy. Baltimore, MD: Johns Hopkins University Press

Barro, R. J., & Martin, X. S. I. (1992). Public Finance in Models of Economic Growth. *Review of Economic Studies*, 59(4), 645-661. https://doi.org/10.2307/2297991

Barro, R.J., & Martin, X. S. I. (2004). *Economic Growth. Second Edition*. Cambridge, MA: MIT Press.

Bucci, A., & Bo, Ch. (2011). On the interaction between public and private capital in economic growth. *Journal of Economics*, 106(2), 133-152. https://doi.org/10.1007/s00712-011-0239-3

Campbell, J., Diamond, P., & Shoven, J. (2001). Estimating the Real Rate of Return on Stocks Over the Long Term. Washington, DC: Social Security Advisory Board.

Chen, B. L., (2007). Factor taxation and labor supply in a dynamic one-sector growth model. *Journal of Economic Dynamics and Control*, 31(12), 3941-3964. https://doi.org/10.1016/j.jedc.2007.01.027

Cichy, K. (2008). Kapitał ludzki i postęp techniczny jako determinanty wzrostu gospodarczego [Human capital and technological progress as determinants of economic growth]. Warszawa: Instytut Wiedzy i Innowacji.

Dasgupta, D. (1999). Growth versus welfare in a model of nonrival infrastructure. *Journal of Development Economics*, 58(2), 359-385. https://doi.org/10.1016/s0304-3878(98)00127-8

Dhont, T., & Heylen, F. (2009). Employment and growth in Europe and the US – the role of fiscal policy composition. *Oxford Economic Papers*, 61(3), 538-565. https://doi.org/10.1093/oep/gpn034

Dobbs, R., Pohl, H., Lin, D-Y., Mischke, J., Garemo, N., Hexter, J.,…, Nanavatty, R. (2013). *Infrastructure productivity: how to save $1 trillion a year*. McKinsey Global Institute.

Easterly W., & Rebelo S. (1993). Fiscal Policy and Economic Growth. *Journal of Monetary Economics*, 32(3), 417-458.

Elmendorf, D. W., & Mankiw, N. G. (1998, March). Government Debt (Working Paper No. 6470). National Bureau of Economic Research. Available at https://ssrn.com/abstract=226215

Escobar-Posada, R. A., & Monteiro, G. (2015). Long-run growth and welfare in a two sector endogenous growth model with productive and non-productive government expenditure. *Journal of Macroeconomics*, 46, 218-234. https://doi.org/10.1016/j.jmacro.2015.09.008

Eurostat. (2010). *Taxation trends in the European Union - Data for the EU Member States, Iceland and Norway*. Retrieved from https://ec.europa.eu/taxation_customs/sites/taxation/files/resources/documents/taxation/gen_info/economic_analysis/tax_structures/2010/2010_full_text_en.pdf

Fisher, W. H., & Turnovsky, S. J. (1998). Public investment, congestion and private capital accumulation. *Economic Journal*, 108(447), 399-413. https://doi.org/10.1111/1468-0297.00294

Futagami, K., Morita, Y., & Shibata, A., (1993). Dynamic analysis of an endogenous growth model with public capital. *Scandinavian Journal of Economics*, 95(4), 607-625. https://doi.org/10.2307/3440914

Gupta, S., Kangur, A., Papageorgiou, C., & Wane, A. (2014). Efficiency-adjusted public capital and growth. *World Development*, 57, 164-178. https://doi.org/10.1016/j.worlddev.2013.11.012

International Monetary Fund. (2015, June). Making public investment more efficient. Retrieved from http://www.imf.org/external/np/pp/eng/2015/061115.pdf

International Monetary Fund. (2016). Investment and Capital Stock Dataset. Available at http://www.imf.org/external/np/fad/publicinvestment/

Kamps, C. (2006). New Estimates of Government Net Capital Stocks for 22 OECD Countries, 1960–2001. *IMF Staff Papers*, 53(1), 120–50.

Kollias, C., & Paleologou, S. M. (2013). Guns, highways and economic growth in the United States. *Economic Modelling*, 30, 449-455. https://doi.org/10.1016/j.econmod.2012.09.048

Konopczyński, M. (2014). How Taxes and Spending on Education Influence Economic Growth in Poland. *Contemporary Economics*, 8(3), 329-348. https://doi.org/10.5709/ce.1897-9254.149

Konopczyński, M. (2015). Optymalna polityka fiskalna w gospodarce otwartej w świetle teorii endogenicznego wzrostu gospodarczego [Optimal fiscal policy in an open economy in the light of the theory of endogenous growth]. Poznań: Wydawnictwo Uniwersytetu Ekonomicznego w Poznaniu.

Lee, Y., & Gordon, R. (2005). Tax structure and economic growth. *Journal of Public Economics*, 89(5-6), 1027-1043. https://doi.org/10.1016/j.jpubeco.2004.07.002

Mankiw, G. N., Romer, D., & Weil, D. N. (1992). A Contribution to the Empirics of Economic Growth. *Quarterly Journal of Economics*, 107(2), 407-437.

Manuelli, R., & Seshadri, A. (2005, June). *Human Capital and the Wealth of Nations*. Paper presented at the meeting of the Society for Economic Dynamics, Budapest, Hungary.

Marrero, G. A., & Novales, A. (2007). Income taxes, public investment and welfare in a growing economy. *Journal of Economic Dynamics and Control*, *31*(10), 3348-3369. https://doi.org/10.1016/j.jedc.2006.10.007

Próchniak, M. (2013). To What Extent Is the Institutional Environment Responsible for Worldwide Differences in Economic Development. *Contemporary Economics*, 7(3), 17-38. https://doi.org/10.5709/ce.1897-9254.87

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy* 94(5), 1002-1037. https://doi.org/10.1086/261420

Turnovsky, S. J. (1997). Fiscal Policy in a growing economy with public capital. *Macroeconomic Dynamics*, 1(3), 615-39. https://doi.org/10.1017/s1365100597004045

Turnovsky, S. J. (2004). The transitional dynamics of fiscal policy: long-run capital accumulation and growth. *Journal of Money, Credit, and Banking*, 36, 883-910. https://doi.org/10.1353/mcb.2004.0069

Turnovsky, S. J. (2009). *Capital Accumulation and Economic Growth in a Small Open Economy*. Cambridge, UK: Cambridge University Press.

Zhang, L., Ru, Y., & Li, J. (2016). Optimal tax structure and public expenditure composition in a simple model of endogenous growth. *Economic Modelling*, 59, 352-360. https://doi.org/10.1016/j.econmod.2016.08.005

Acknowledgements

This research has been supported by the Polish National Science Center; grant number 2017/25/B/HS4/02076. The author would like to thank two anonymous referees for their helpful suggestions.

Appendix

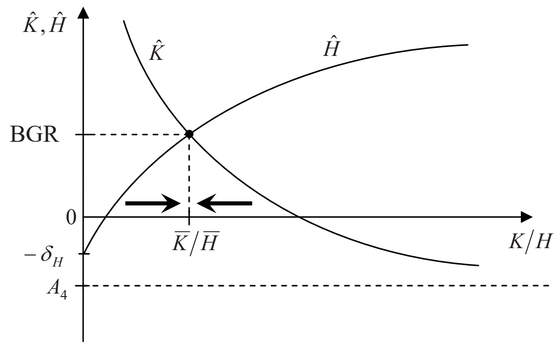

Figure A1. Graphs of the $\hat{K}(K/H)$ and $\hat{H}(K/H)$ functions

Proof of Proposition 1.
By definition, $P = bK_P/Y$, thus $\dot{P} = b\dfrac{\dot{K}_P Y - K_P \dot{Y}}{Y^2} = b\left(\dfrac{\dot{K}_P}{Y} - \dfrac{K_P}{Y}\hat{Y}\right)$. Recall that $\dot{K}_P = G_K - \delta_P K_P = \gamma_P Y - \delta_P K_P$; therefore, $\dot{P} = b\left(\gamma_P - \delta_P \dfrac{K_P}{Y} - \dfrac{K_P}{Y}\hat{Y}\right)$, which can be written in following equivalent form:

$$\dot{P} = b\gamma_P - (\hat{Y} + \delta_P)P. \qquad \text{(A0)}$$

For any given (positive) value of b, δ_P, and $\hat{Y}$, equation (A0) constitutes a linear differential equation of the form $\dot{P} = f(P)$. Figure A0 presents the phase diagram of this equation. Regardless of the initial value of $P(t=0) > 0$, over time, $P \rightarrow \dfrac{b\gamma_P}{\hat{Y} + \delta_P}$.

Proof of Proposition 2.
Assume that $P = const. > 0$. This assumption allows for proceeding along the lines of the proofs of Propositions 1 and 2 in Konopczyński (2014). Recall that the rates of growth of private capital and human capital are given by equations (36) and (37). Let us determine the signs of all expressions that are marked with symbols A_i. Under the assumptions adopted regarding the signs and the values of tax rates, rates of savings, and other parameters, it can be shown that:

$$A_2, A_3, A_5, A_6 > 0,\ A_4 < 0 \ \text{ and } \ A_2 < 1,\ A_6 < 1. \qquad \text{(B1)}$$

Note that, for a given (constant) value of P, we can treat $\hat{K}$ and $\hat{H}$ as functions of a single variable K/H. These functions are given by equations (36) and (37) and have identical properties to their counterparts in Konopczyński (2014). From (B1), the function $\hat{K}(K/H)$ is decreasing and strictly convex. Moreover, $\hat{K} \xrightarrow[K/H \to 0^+]{} +\infty$, and $\hat{K} \xrightarrow[K/H \to +\infty]{} A_4$. The function $\hat{H}(K/H)$ is increasing, strictly concave, $\hat{H}(K/H = 0) = -\delta_H$, and $\hat{H} \xrightarrow[K/H \to +\infty]{} +\infty$. The graphs of these functions are illustrated in figure A1. Due to the properties of these functions, there is exactly one point of intersection, i.e., there exists exactly one ratio $\overline{K}/\overline{H}$ for which $\hat{K} = \hat{H}$. The values of both functions at this point determine the balanced growth rate (the BGR). The balanced growth state is globally asymptotically stable, as shown in figure A1. In equilibrium, $\hat{K} = \hat{H}$, which, together with (3), implies that $\hat{Y} = \hat{K} = \hat{H}$.

We proved that, *for any given (constant) value of P*, the economy converges to the balanced growth state where $\hat{Y} = \hat{K} = \hat{H}$, which is unique and globally asymptotically stable. Proposition 1 implies that, along the BGR path, $P \rightarrow \frac{b\gamma_P}{\hat{Y} + \delta_P} = const.$ These two facts imply that there is a unique balanced growth equilibrium and that it is globally asymptotically stable. In the steady state:

$$\overline{P} = \frac{b\gamma_P}{\hat{Y} + \delta_P} = const. > 0 \tag{B0}$$